



ME1251 THERMAL ENGINEERING

UNIT IV

AIR COMPRESSORS

www.mechanical.in

CONTENTS

TECHNICAL TERMS

4.1 Classification of compressors

4.2 Positive Displacement compressors

4.2.1 Double acting compressor

4.2.2 Diaphragm Compressors

4.3 Rotary compressors

4.3.1 Lobe compressor

4.3.2 Liquid ring compressor

4.3.3 Vane Type compressor:

4.3.4 Screw Type compressor

4.3.5 Scroll Type Compressor

4.4 Non-Positive displacement compressors

4.4.1 Centrifugal Compressor

4.4.2 Axial Compressor

4.4.3 Roots Blower Compressor

4.5 Multistage Compression

4.5.1 Advantages of Multi-stage compression

4.6 Work done in a single stage reciprocating compressor without clearance volume

4.6.1 Work done in a single stage reciprocating compressor with clearance volume

4.7 Volumetric Efficiency

4.7.1 Mathematical analysis of multistage compressor is done with following assumptions

4.8 Solved Problems

4.9 Two-Marks University Questions

4.10 University Essay Questions

TECHNICAL TERMS

1. Volumetric Efficiency of the Compressor

It is the ratio of actual volume of air drawn in the compressor to the stroke volume of the compressor.

2. Mechanical efficiency

It is the ratio of indicated power to shaft power or brake power of motor

3. Isentropic efficiency

It is the ratio of the isentropic power to the brake power required to drive the compressor.

4. Centrifugal compressor

The flow of air is perpendicular to the axis of compressor

5. Axial flow compressor

The flow of air is parallel to the axis of compressor

6. Compression:

The process of increasing the pressure of air, gas and vapour by reducing its volume is called as compression.

7. Single acting compressor:

The suction, compression and the delivery of air takes on the one side of piston

8. Double acting compressor:

The suction, compression and the delivery of air takes place on both sides of the piston.

9. Multi stage compressor:

The compression of air from initial pressure to the final pressure is carried out in more than one cylinder.

10. Application of compressed air:

Pneumatic brakes, drills, jacks, lifts, spray of paintings, shop cleaning, injecting the fuel in diesel engine, supercharging, refrigeration and in air conditioning systems.

11. Inter cooler:

It is a simple heat exchanger, exchanges the heat of compressed air from low pressure compressor to circulating water before the air enters to high pressure compressor. The purpose of intercooling is to minimize the work of compression.

12. Isentropic efficiency:

It is the ratio of isentropic power to the brake power required to drive the compressor.

13. Clearance ratio:

It is the ratio of clearance volume to the swept volume or stroke volume is called as clearance ratio.

14. Isothermal efficiency:

It is the ratio between isothermal work to the actual work of the compressor.

15. Compression ratio:

The ratio between total volume and the clearance volume of the cylinder is called compression ratio.

16. Perfect intercooling:

When the temperature of the air leaving the intercooler is equal to the original atmospheric air temperature, then the inter cooling is called perfect intercooling.

UNIT-IV

AIR COMPRESSORS

4.1 Classification of compressors:

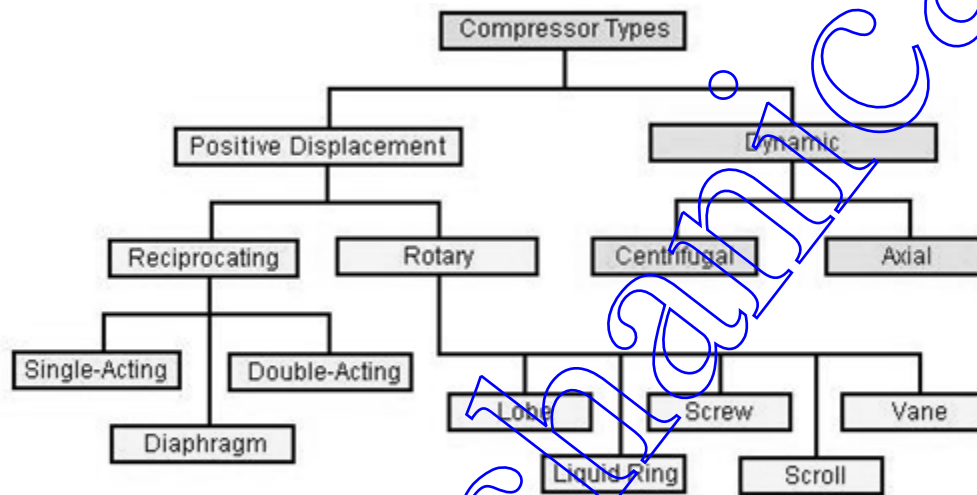


Fig 1.

The compressors are also classified based on other aspects like

1. Number of stages (single-stage, 2-stage and multi-stage),
2. Cooling method and medium (Air cooled, water cooled and oil-cooled),
3. Drive types (Engine driven, Motor driven, Turbine driven, Belt, chain, gear or direct coupling drives),
4. Lubrication method (Splash lubricated or forced lubrication or oil-free compressors).
5. Service Pressure (Low, Medium, High)

4.2 Positive Displacement compressors: Reciprocating Compressor: Single-Acting Reciprocating compressor:

These are usually reciprocating compressors, which has piston working on air only in one direction. The other end of the piston is often free or open which does not perform any work. The

air is compressed only on the top part of the piston. The bottom of the piston is open to crankcase and not utilized for the compression of air.

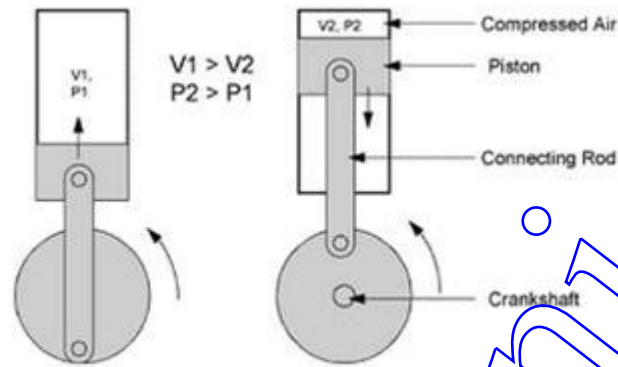


Fig 2 Piston & Cylinder Arrangement

4.2.1 Double acting compressor:

These compressors are having two sets of suction/intake and delivery valves on both sides of the piston. As the piston moves up and down, both sides of the piston is utilized in compressing the air. The intake and delivery valves operate corresponding to the stroke of the compressor. The compressed air delivery is comparatively continuous when compared to a single-acting air compressor. Thus both sides of the pistons are effectively used in compressing the air.

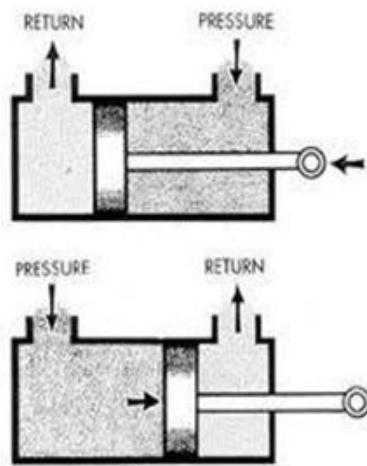


Fig 3

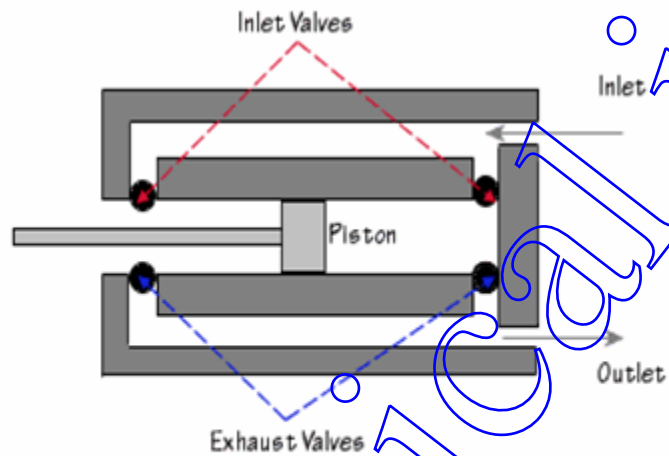


Fig 4 Double Acting Compressor

4.2.2 Diaphragm Compressors: In the diaphragm compressor, the piston pushes against a diaphragm, so the air does not come in contact with the reciprocating parts. This type compressor is preferred for food preparation, pharmaceutical, and chemical industries, because no effluent from the compressor enters the fluid.

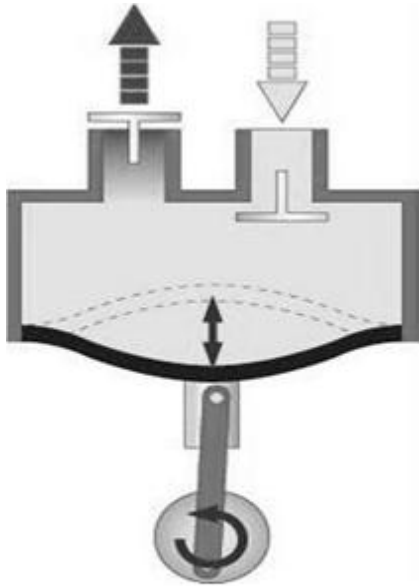


Fig 5 Diaphragm Compressors

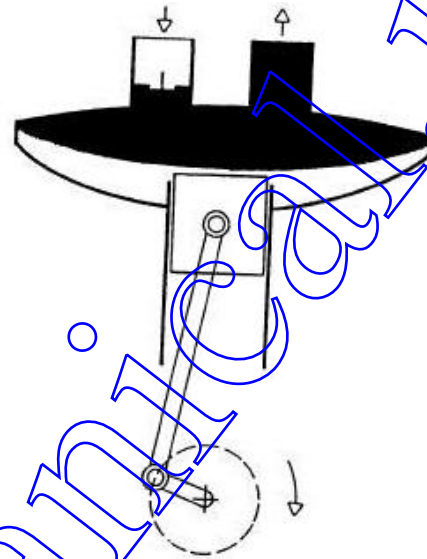


Fig 6 Diaphragm Compressors

4.3 Rotary compressors:

4.3.1 Lobe compressor:

The Lobe type air compressor is very simpler type with no complicated moving parts. There are single or twin lobes attached to the drive shaft driven by the prime mover. The lobes are displaced by 90 degrees. Thus if one of the lobes is in horizontal position, the other at that particular instant will be in vertical position. Thus the air gets trapped in between these lobes and as they rotate they get compressed and delivered to the delivery line.

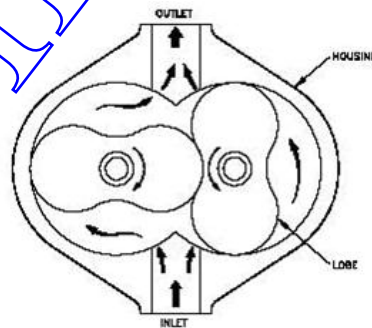


Fig 7 Lobe compressor

4.3.2 Liquid ring compressor:

Liquid ring compressors require a liquid to create a seal. For medical applications, liquid ring compressors are always sealed with water but not oil. An impeller, which is offset so the impeller is not in the center of the pump housing, rotates and traps pockets of air in the space between the impeller fins and the compressor housing. The impeller is typically made of brass. As the impeller turns, there is a pocket of air that is trapped in the space between each of the fins. The trapped air is compressed between the impeller and the pump housing, sealed with the water ring. As the air is compressed, it's then pushed out of the pumps discharge. To avoid possible contaminants the compressor is always getting a supply of fresh sealing water. In a “once through” system, sealing water is drained and used only once, while in a “partial re-circulating” system, some (but never all) of the discharged water is re-circulated.

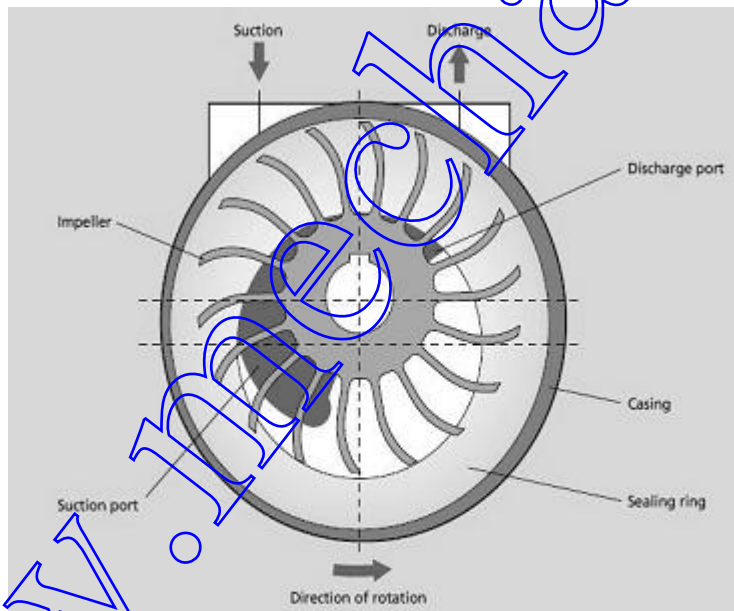


Fig 8 Liquid ring compressor

4.3.3 Vane Type compressor:

The rotary slide vane-type, as illustrated in Figure, has longitudinal vanes, sliding radially in a slotted rotor mounted eccentrically in a cylinder. The centrifugal force carries the sliding vanes against the cylindrical case with the vanes forming a number of individual longitudinal cells in

the eccentric annulus between the case and rotor. The suction port is located where the longitudinal cells are largest. The size of each cell is reduced by the eccentricity of the rotor as the vanes approach the discharge port, thus compressing the air. This type of compressor, looks and functions like a vane type hydraulic pump. An eccentrically mounted rotor turns in a cylindrical housing having an inlet and outlet. Vanes slide back and forth in grooves in the rotor. Air pressure or spring force keeps the tip of these vanes in contact with the housing. Air is trapped in the compartments formed by the vanes and housing and is compressed as the rotor turns.

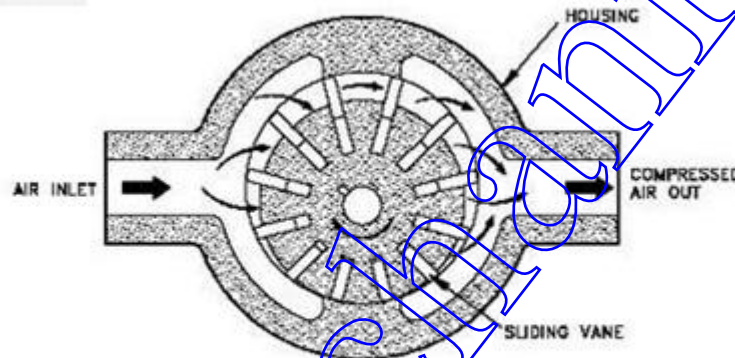


Fig 9 Vane Type compressor

4.3.4 Screw Type compressor:

The screw compressors are efficient in low air pressure requirements. Two screws rotate intermeshing with each other, thus trapping air between the screws and the compressor casing, forming pockets which progressively travel and gets squeezed and delivering it at a higher pressure which opens the delivery valve. The compressed air delivery is continuous and quiet in operation than a reciprocating compressor. Rotary air compressors are positive displacement compressors. The most common rotary air compressor is the single stage helical or spiral lobe oil flooded screw air compressor. These compressors consist of two rotors within a casing where the rotors compress the air internally. There are no valves. These units are basically oil cooled (with air cooled or water cooled oil coolers) where the oil seals the internal clearances. Since the cooling takes place right inside the compressor, the working parts never experience extreme

operating temperatures. The rotary compressor, therefore, is a continuous duty, air cooled or water cooled compressor package.

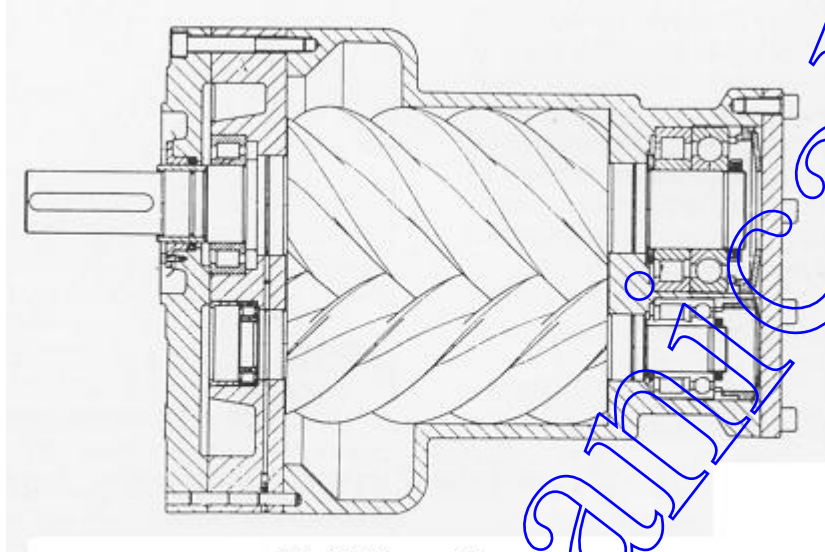


Fig 10 Screw Type compressor

4.3.5 Scroll Type Compressor:

This type of compressor has a very unique design. There are two scrolls that look like loosely rolled up pieces of paper—one rolled inside the other. The orbiting scroll rotates inside of the stationary scroll. The air is forced into progressively smaller chambers towards the center. The compressed air is then discharged through the center of the fixed scroll. No inlet or exhaust valves are needed.

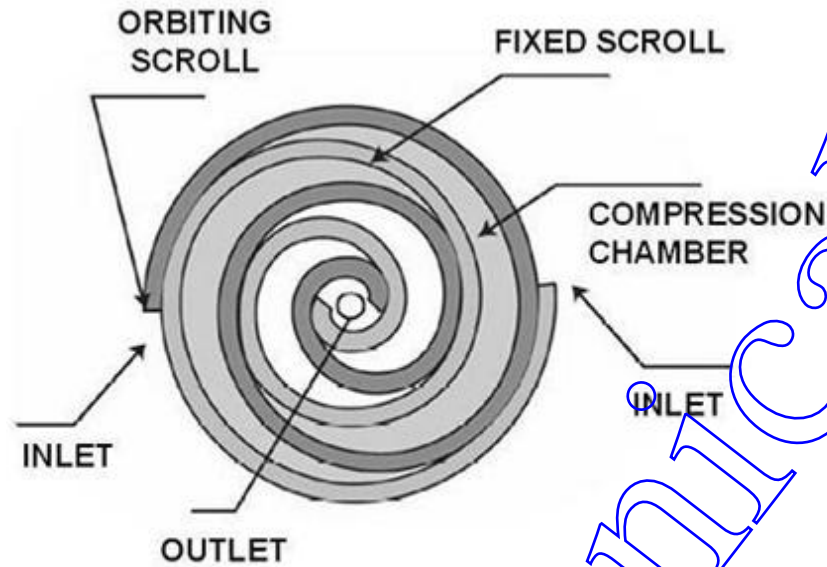


Fig 11 Scroll Type Compressor

4.4 Non-Positive displacement compressors or Dynamic compressor:

4.4.1 Centrifugal Compressor:

The centrifugal air compressor is a **dynamic** compressor which depends on transfer of energy from a **rotating impeller** to the air. Centrifugal compressors produce high-pressure discharge by converting angular momentum imparted by the rotating impeller (dynamic displacement). In order to do this efficiently, centrifugal compressors rotate at higher speeds than the other types of compressors. These types of compressors are also designed for higher capacity because flow through the compressor is continuous. Adjusting the inlet guide vanes is the most common method to control capacity of a centrifugal compressor. By closing the guide vanes, volumetric flows and capacity are reduced.

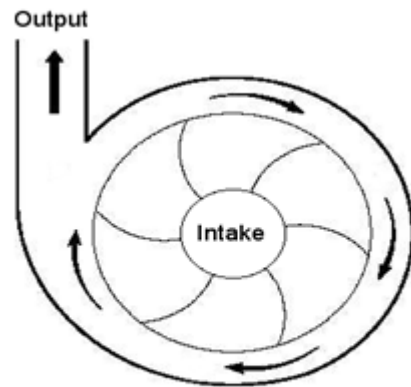


Fig 12 Centrifugal Compressor

The centrifugal air compressor is an oil free compressor by design. The oil lubricated running gear is separated from the air by shaft seals and atmospheric vents. The centrifugal air compressor is a dynamic compressor which depends on a rotating impeller to compress the air. In order to do this efficiently, centrifugal compressors must rotate at higher speeds than the other types of compressors. These types of compressors are designed for higher capacity because flow through the compressor is continuous and oil free by design.

4.4.2 Axial Compressor:

These are similar to centrifugal compressors except the direction of air flow is axial. The blades of the compressor are mounted onto the hub and in turn onto the shaft. As the shaft rotates at a high speed, the ambient air is sucked into the compressor and then gets compressed (high speed of rotation of the blades impart energy to the air) and directed axially for further usage. An axial flow compressor, in its very simple form is called as axial flow fan, which is commonly used for domestic purposes. The pressure built depends on the number of stages. These are commonly used as vent fans in enclosed spaces, blower ducts, etc. One can find its main application in the aerospace industry, where the gas turbines drive the axial flow air compressors.

4.4.3 Roots Blower Compressor:

This type is generally called as blower. The discharge air pressure obtained from this type of machine is very low. The Discharge Pressure of 1 bar can be obtained in Single Stage and pressure of 2.2 bar is obtained from Stage. The discharge pressure achieved by two rotors which

have separate parallel axis and rotate in opposite directions. This is the example of Positive Displacement Compressor in Rotary Type Air Compressor.

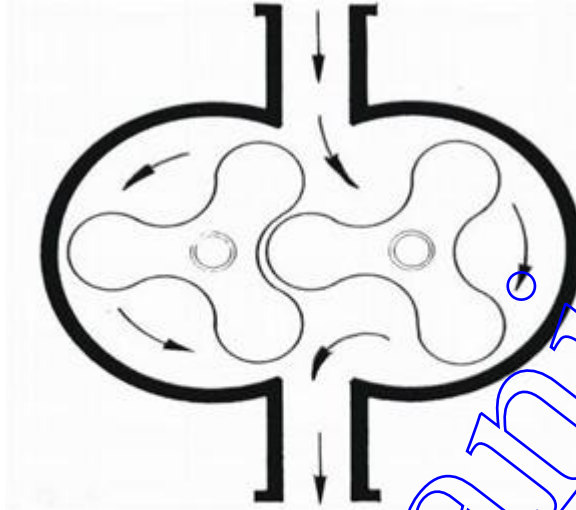


Fig 13 Roots Blower Compressor

4.5 Multistage Compression:

Multistage compression refers to the compression process completed in more than one stage i.e., a part of compression occurs in one cylinder and subsequently compressed air is sent to subsequent cylinders for further compression. In case it is desired to increase the compression ratio of compressor then multi-stage compression becomes inevitable. If we look at the expression for volumetric efficiency then it shows that the volumetric efficiency decreases with increase in pressure ratio. This aspect can also be explained using p-V representation shown in Figure.

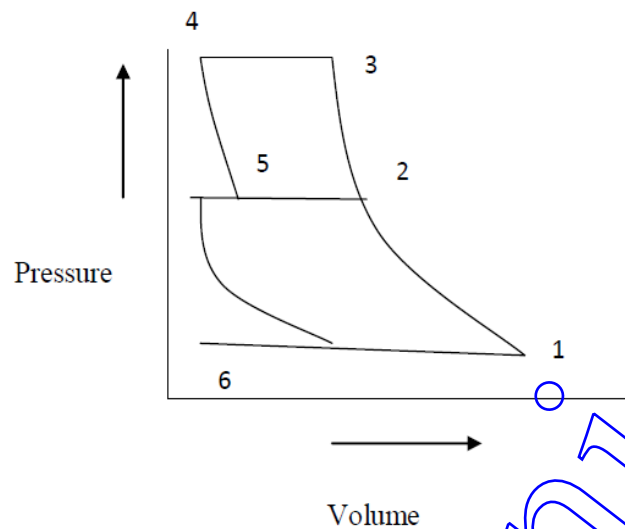


Fig 14 PV Diagram

A multi-stage compressor is one in which there are several cylinders of different diameters. The intake of air in the first stage gets compressed and then it is passed over a cooler to achieve a temperature very close to ambient air. This cooled air is passed to the intermediate stage where it is again getting compressed and heated. This air is again passed over a cooler to achieve a temperature as close to ambient as possible. Then this compressed air is passed to the final or the third stage of the air compressor where it is compressed to the required pressure and delivered to the air receiver after cooling sufficiently in an after-cooler.

4.5.1 Advantages of Multi-stage compression:

1. The work done in compressing the air is reduced, thus power can be saved
2. Prevents mechanical problems as the air temperature is controlled
3. The suction and delivery valves remain in cleaner condition as the temperature and vaporization of lubricating oil is less
4. The machine is smaller and better balanced
5. Effects from moisture can be handled better, by draining at each stage
6. Compression approaches near isothermal
7. Compression ratio at each stage is lower when compared to a single-stage machine

4.6 Work done in a single stage reciprocating compressor without clearance volume:

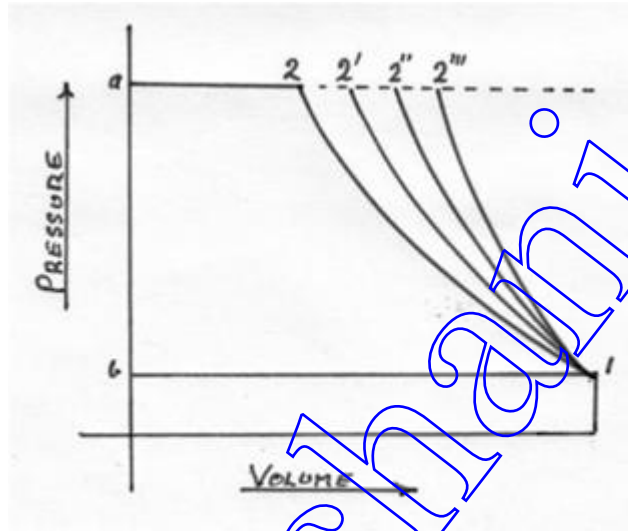


Fig 15 PV Diagram

Air enters compressor at pressure p_1 and is compressed upto p_2 . Compression work requirement can be estimated from the area below the each compression process. Area on p - V diagram shows that work requirement shall be minimum with isothermal process 1-2''. Work requirement is maximum with process 1-2 ie., adiabatic process. As a designer one shall be interested in a compressor having minimum compression work requirement. Therefore, ideally compression should occur isothermally for minimum work input. In practice it is not possible to have isothermal compression because constancy of temperature during compression can not be realized. Generally, compressors run at substantially high speed while isothermal compression requires compressor to run at very slow speed so that heat evolved during compression is dissipated out and temperature remains constant. Actually due to high speed running of compressor the compression process may be assumed to be near adiabatic or polytropic process



following law of compression as $Pv^n=C$ with of „n“ varying between 1.25 to 1.35 for air. Compression process following three processes is also shown on T-s diagram.

It is thus obvious that actual compression process should be compared with isothermal compression process. A mathematical parameter called isothermal efficiency is defined for quantifying the degree of deviation of actual compression process from ideal compression process. Isothermal efficiency is defined by the ratio is isothermal work and actual indicated work in reciprocating compressor.

$$\text{Isothermal efficiency} = \frac{\text{Isothermal work}}{\text{Actual indicated Work}}$$

Practically, compression process is attempted to be closed to isothermal process by air/water cooling, spraying cold water during compression process. In case of multistage compression process the compression in different stages is accompanied by intercooling in between the stages.

$P_2 V_2$

Mathematically, for the compression work following polytropic process, $PV^n=C$. Assuming negligible clearance volume the cycle work done.

$W_c = \text{Area on p-V diagram}$

$$W_c = \left[p_2 V_2 + \left(\frac{p_2 V_2 - p_1 V_1}{n-1} \right) \right] - p_1 V_1$$

$$= \left[\left(\frac{n}{n-1} \right) [p_2 V_2 - p_1 V_1] \right]$$

$$= \left(\frac{n}{n-1} \right) (p_1 V_1) \left[\frac{p_2 V_2}{p_1 V_1} - 1 \right]$$

$$= \left(\frac{n}{n-1} \right) (p_1 V_1) \left[\left(\frac{p_2}{p_1} \right)^{\left(\frac{n-1}{n} \right)} - 1 \right]$$

$$= \left(\frac{n}{n-1} \right) (mRT_1) \left[\left(\frac{p_2}{p_1} \right)^{\left(\frac{n-1}{n} \right)} - 1 \right]$$

$$= \left(\frac{n}{n-1} \right) (mR)(T_2 - T_1)$$

In case of compressor having isothermal compression process, $n = 1$, i.e., $p_1 V_1 = p_2 V_2$

$$W_{iso} = p_2 V_2 + p_1 V_1 \ln r - p_1 V_1$$

$$W_{iso} = p_1 V_1 \ln r, \quad \text{where, } r = \frac{V_1}{V_2}$$

In case of compressor having adiabatic compression process,

$$W_{adiabatic} = \left(\frac{\gamma}{\gamma - 1} \right) (mR)(T_2 - T_1) \quad (\text{Or})$$

$$W_{adiabatic} = (mC_p)(T_2 - T_1)$$

$$W_{adiabatic} = (m)(h_2 - h_1)$$

$$\eta_{iso} = \frac{p_1 V_1 \ln r}{\left(\frac{n}{n-1} \right) (p_1 V_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]}$$

The isothermal efficiency of a compressor should be close to 100% which means that actual compression should occur following a process close to isothermal process. For this the mechanism be derived to maintain constant temperature during compression process. Different arrangements which can be used are:

- (i) Faster heat dissipation from inside of compressor to outside by use of fins over cylinder. Fins facilitate quick heat transfer from air being compressed to atmosphere so that temperature rise during compression can be minimized.
- (ii) Water jacket may be provided around compressor cylinder so that heat can be picked by cooling water circulating through water jacket. Cooling water circulation around compressor regulates rise in temperature to great extent.
- (iii) The water may also be injected at the end of compression process in order to cool the air being compressed. This water injection near the end of compression process requires special arrangement in compressor and also the air gets mixed with water and needs to be separated out

before being used. Water injection also contaminates the lubricant film inner surface of cylinder and may initiate corrosion etc, the water injection is not popularly used.

(iv) In case of multistage compression in different compressors operating serially, the air leaving one compressor may be cooled up to ambient state or somewhat high temperature before being injected into subsequent compressor. This cooling of fluid being compressed between two consecutive compressors is called inter cooling and is frequently used in case of multistage compressors.

4.6.1 Work done in a single stage reciprocating compressor with clearance volume:

Considering clearance volume: With clearance volume the cycle is represented on Figure. The work done for compression of air polytropically can be given by the area enclosed in cycle 1-2-3-4. Clearance volume in compressors varies from 1.5% to 35% depending upon type of compressor.

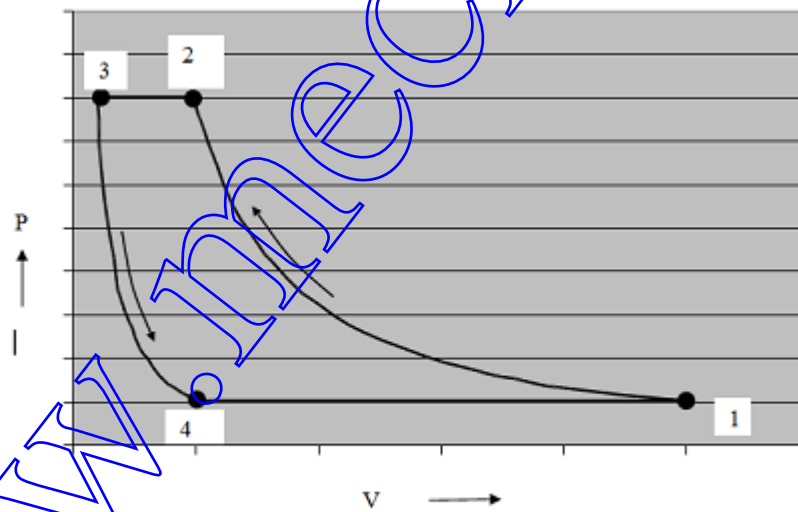


Fig16 PV Diagram

$$W_{c,with CV} = \text{Area 1234}$$

$$= \left(\frac{n}{n-1}\right)(p_1 V_1) \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right] - \left(\frac{n}{n-1}\right)(p_4 V_4) \left[\left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$\text{Here } P_1 = P_4, P_2 = P_3$$

$$= \left(\frac{n}{n-1}\right)(p_1 V_1) \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right] - \left(\frac{n}{n-1}\right)(p_1 V_4) \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \left(\frac{n}{n-1}\right)(p_1) \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right] (V_1 - V_4)$$

In the cylinder of reciprocating compressor (V1-V4) shall be the actual volume of air delivered per cycle. $V_d = V_1 - V_4$. This (V1 - V4) is actually the volume of air inhaled in the cycle and delivered subsequently.

$$W_{c,with CV} = \left(\frac{n}{n-1}\right)(p_1 V_d) \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

If air is considered to behave as perfect gas then pressure, temperature, volume and mass can be inter related using perfect gas equation. The mass at state 1 may be given as m_1 mass at state 2 shall be m_1 , but at state 3 after delivery mass reduces to m_2 and at state 4 it shall be m_2 .

$$\text{So, at state 1, } p_1 V_1 = m_1 R T_1$$

$$\text{at state 2, } p_2 V_2 = m_1 R T_2$$

$$\text{at state 3, } p_3 V_3 = m_2 R T_3 \text{ or } p_2 V_3 = m_2 R T_3$$

$$\text{at state 4, } p_4 V_4 = m_2 R T_4 \text{ or } p_1 V_4 = m_2 R T_4$$

Ideally there shall be no change in temperature during suction and delivery

i.e. $T_4 = T_1$ and $T_2 = T_3$ from earlier equation

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)(p_1) \left[\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1 \right] (V_1 - V_4)$$

Temperature and pressure can be related as,

$$\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} = \frac{T_2}{T_1} \quad \text{and} \quad \left(\frac{p_4}{p_3}\right)^{\frac{(n-1)}{n}} = \frac{T_4}{T_3} \quad \Rightarrow \quad \left(\frac{p_1}{p_2}\right)^{\frac{(n-1)}{n}} = \frac{T_4}{T_3}$$

Substituting

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)(m_1RT_1 - m_2RT_4) \left[\frac{T_2}{T_1} - 1 \right]$$

Substituting for constancy of temperature during suction and deliver

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)(m_1RT_1 - m_2RT_1) \left[\frac{T_2 - T_1}{T_1} \right]$$

Or

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)(m_1 - m_2)R(T_2 - T_1)$$

Thus $(m_1 - m_2)$ denotes the mass of air sucked or delivered. For unit mass of air delivered the work done per kg of air can be given as,

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)R(T_2 - T_1) \quad \text{per kg of air}$$

Thus from above expressions it is obvious that the clearance volume reduces the effective swept volume i.e., the mass of air handled but the work done per kg of air delivered remains unaffected. From the cycle work estimated as above the theoretical power required for running compressor shall be,

For single acting compressor running with N rpm, power input required, assuming clearance volume.

$$\text{Power required} = \left[\left(\frac{n}{n-1} \right) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] p_1 (V_1 - V_4) \right] (N)$$

For double acting compressor, Power

$$\text{Power required} = \left[\left(\frac{n}{n-1} \right) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] p_1 (V_1 - V_4) \right] (2N)$$

4.7 Volumetric Efficiency:

Volumetric efficiency of compressor is the measure of the deviation from volume handling capacity of compressor. Mathematically, the volumetric efficiency is given by the ratio of actual volume of air sucked and swept volume of cylinder. Ideally the volume of air sucked should be equal to the swept volume of cylinder, but it is not so in actual case. Practically the volumetric efficiency lies between 60 to 90%. Volumetric efficiency can be overall volumetric efficiency and absolute volumetric efficiency as given below.

$$\text{Overall volumetric efficiency} = \frac{\text{Volume of free air sucked in cylinder}}{\text{Swept volume of LP cylinder}}$$

$$(\text{Volumetric efficiency})_{\text{freeaircondition}} = \frac{\text{Volume of free air sucked in cylinder}}{(\text{Swept volume of LP cylinder})_{\text{freeaircondition}}}$$

Here free air condition refers to the standard conditions. Free air condition may be taken as 1 atm or 1.01325 bar and 15°C or 288K. consideration for free air is necessary as otherwise the different compressors can not be compared using volumetric efficiency because specific volume or density of air varies with altitude. It may be seen that a compressor at datum level (sea level) shall deliver large mass than the same compressor at high altitude.

This concept is used for giving the capacity of compressor in terms of „free air delivery“ (FAD). “Free air delivery is the volume of air delivered being reduced to free air conditions”. In case of air the free air delivery can be obtained using perfect gas equation as,

$$\frac{p_a V_a}{T_a} = \frac{p_1 (V_1 - V_4)}{T_1} = \frac{p_2 (V_2 - V_3)}{T_2}$$

Where subscript a or pa, Va, Ta denote properties at free air conditions

$$V_a = \frac{p_1 T_a}{p_a T_1} (V_1 - V_4) = \text{FAD per cycle}$$

This volume Va gives „free air delivered“ per cycle by the compressor. Absolute volumetric efficiency can be defined, using NTP conditions in place of free air conditions.

$$\eta_{vol} = \frac{\text{FAD}}{\text{Swept volume}} = \frac{V_a}{(V_1 - V_2)} = \frac{p_1 T_1 (V_1 - V_4)}{p_a T_a (V_1 - V_2)}$$

$$\eta_{vol} = \left(\frac{p_1 T_a}{p_a T_1} \right) \left\{ \frac{(V_s + V_c) - V_4}{V_s} \right\}$$

Here Vs is the swept volume = V1 – V3 and Vc is the clearance volume = V3

$$\eta_{vol} = \left(\frac{p_1 T_a}{p_a T_1} \right) \left\{ 1 + \left(\frac{V_c}{V_s} \right) - \left(\frac{V_4}{V_s} \right) \right\}$$

$$\text{Here } \frac{V_4}{V_s} = \frac{V_4}{V_c} \cdot \frac{V_c}{V_s} = \left(\frac{V_4}{V_3} \cdot \frac{V_c}{V_s} \right)$$

$$\eta_{vol} = \left(\frac{p_1 T_a}{p_a T_1} \right) \left\{ 1 + C - C \left(\frac{V_4}{V_3} \right) \right\}$$

$$\eta_{vol} = \left(\frac{p_1 T_a}{p_a T_1} \right) \left\{ 1 + C - C \left(\frac{p_2}{p_1} \right)^{\frac{1}{n}} \right\}$$

Volumetric efficiency depends on ambient pressure and temperature, suction pressure and temperature, ratio of clearance to swept volume, and pressure limits. Volumetric efficiency increases with decrease in pressure ratio in compressor.

4.7.1 Mathematical analysis of multistage compressor is done with following assumptions:

- (i) Compression in all the stages is done following same index of compression and there is no pressure drop in suction and delivery pressures in each stage. Suction and delivery pressure remains constant in the stages.
- (ii) There is perfect inter cooling between compression stages.
- (iii) Mass handled in different stages is same i.e., mass of air in LP and HP stages are same.
- (iv) Air behaves as perfect gas during compression.

From combined p-V diagram the compressor work requirement can be given as,

$$\text{Work requirement in LP cylinder, } W_{LP} = \left(\frac{n}{n-1} \right) P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{(n-1)}{n}} - 1 \right]$$

$$\text{Work requirement in HP cylinder, } W_{HP} = \left(\frac{n}{n-1} \right) P_2 V_2 \left[\left(\frac{P_2}{P_1} \right)^{\frac{(n-1)}{n}} - 1 \right]$$

For perfect intercooling, $p_1 V_1 = p_2 V_2$ and

$$W_{HP} = \left(\frac{n}{n-1} \right) P_2 V_2 \left[\left(\frac{P_2}{P_1} \right)^{\frac{(n-1)}{n}} - 1 \right]$$

Therefore, total work requirement, $W_c = W_{LP} + W_{HP}$, for perfect inter cooling

$$W_c = \left(\frac{n}{n-1} \right) \left[P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{(n-1)}{n}} - 1 \right] + P_2 V_2 \left[\left(\frac{P_2}{P_1} \right)^{\frac{(n-1)}{n}} - 1 \right] \right]$$

$$= \left(\frac{n}{n-1} \right) \left[P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] + P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] \right]$$

$$W_c = \left(\frac{n}{n-1} \right) P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} + \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 2 \right]$$

Minimum work required in two stage compressor:

Minimum work required in two stage compressor can be given by

$$W_{c,min} = \left(\frac{n}{n-1} \right) P_1 V_1 \cdot 2 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

For I number of stages, minimum work.

$$W_{c,min} = i \cdot \left(\frac{n}{n-1} \right) P_1 V_1 \left[\left(\frac{P_{i+1}}{P_1} \right)^{\frac{n-1}{ni}} - 1 \right]$$

It also shows that for optimum pressure ratio the work required in different stages remains same for the assumptions made for present analysis. Due to pressure ratio being equal in all stages the temperature ratios and maximum temperature in each stage remains same for perfect inter cooling. If the actual volume sucked during suction stroke is $V_1, V_2,$ and $V_3 \dots$ For different stages they by perfect gas law, $P_1 V_1 = RT_1, P_2 V_2 = RT_2, P_3 V_3 = RT_3$ For perfect inter cooling $P_1 V_1 = RT_1, P_2 V_2 = RT_1, P_3 V_3 = RT_1$ $P_1 V_1 = P_2 V_2 = RT_2, P_3 V_3 = \dots$

(If the volumetric efficiency of respective stages in $\eta_{V_1}, \eta_{V_2}, \eta_{V_3}, \dots$)

Then theoretical volume of cylinder1, $V_{1,th} = \frac{V_1}{\eta_{V1}}; V_1 = \eta_{V1} \cdot V_{1,th}$

Cylinder 2, $V_{2,th} = \frac{V_2}{\eta_{V2}}; V_2 = \eta_{V2} \cdot V_{2,th}$

Cylinder 3, $V_{3,th} = \frac{V_3}{\eta_{V3}}; V_3 = \eta_{V3} \cdot V_{3,th}$

Substituting,

$$P_1 \cdot \eta_{V1} \cdot V_{1,th} = P_2 \cdot \eta_{V2} \cdot V_{2,th} = P_3 \cdot \eta_{V3} \cdot V_{3,th} = \dots$$

Theoretical volumes of cylinder can be given using geometrical dimensions of cylinder as diameters $D_1, D_2, D_3 \dots$ and stroke lengths $L_1, L_2, L_3 \dots$

Or
$$V_{1,th} = \frac{\pi}{4} \cdot D_1^2 \cdot L_1$$

$$V_{2,th} = \frac{\pi}{4} \cdot D_2^2 \cdot L_2$$

$$V_{3,th} = \frac{\pi}{4} \cdot D_3^2 \cdot L_3$$

Or
$$P_1 \cdot \eta_{V1} \cdot \frac{\pi}{4} \cdot D_1^2 \cdot L_1 = P_2 \cdot \eta_{V2} \cdot \frac{\pi}{4} \cdot D_2^2 \cdot L_2$$

$$P_3 \cdot \eta_{V3} \cdot \frac{\pi}{4} \cdot D_3^2 \cdot L_3 = \dots$$

$$P_1 \cdot \eta_{V1} \cdot \frac{\pi}{4} \cdot D_1^2 \cdot L_1 = P_2 \cdot \eta_{V2} \cdot \frac{\pi}{4} \cdot D_2^2 \cdot L_2$$

$$= P_3 \cdot \eta_{V3} \cdot D_3^2 \cdot L_3 = \dots$$

If the volumetric efficiency is same for all cylinders, i.e. $\eta_{V1} = \eta_{V2} = \eta_{V3} = \dots$ and stroke for all cylinder is same i.e. $L_1 = L_2 = L_3 = \dots$

Then, $D_1^2 P_1 = D_2^2 P_2 = D_3^2 P_3 = \dots$

These generic relations may be used for getting the ratio of diameters of cylinders of multistage compression.

Energy balance: Energy balance may be applied on the different components constituting multistage compression.

For LP stage the steady flow energy equation can be written as below:

$$m \cdot h_1 + W_{LP} = m \cdot h_2 + Q_{LP}$$

$$Q_{LP} = W_{LP} - m(h_2 - h_1)$$

$$Q_{LP} = W_{LP} - m \cdot C_p (T_2 - T_1)$$

For intercooling between LP and HP stage steady flow energy equation shall be;

$$m \cdot h_2 = m \cdot h_2 + Q_{ht}$$

$$Q_{ht} = m(h_2 - h_2)$$

$$Q_{ht} = m \cdot C_p (T_2 - T_2)$$

For HP stage the steady flow energy equation yields.

$$m \cdot h_2 + W_{HP} = m \cdot h_3 + Q_{HP}$$

$$Q_{HP} = W_{HP} + m(h_2 - h_3)$$

$$Q_{HP} = W_{HP} + m \cdot C_p (T_2 - T_3) = W_{HP} - m \cdot C_p (T_3 - T_2)$$

In case of perfect intercooling and optimum pressure ratio, $T_2 = T_1$ and $T_3 = T_2$.

Hence for these conditions

$$Q_{LP} = W_{LP} - m \cdot C_p (T_2 - T_1)$$

$$Q_{ht} = m \cdot C_p (T_2 - T_1)$$

$$Q_{HP} = W_{HP} - m \cdot C_p (T_3 - T_2)$$

Total heat rejected during compression shall be the sum of heat rejected during compression and heat extracted in intercooler for perfect inter cooling. Heat rejected during compression for polytropic process

$$= \left(\frac{\gamma - n}{\gamma - 1} \right) \times \text{Work}$$

4.8 Solved Problems

1. A single stage double acting air compressor of 150KW power takes air in at 1 bar & delivers at 6 bar. The compression follows the law $PV^{1.35} = C$. the compressor runs at 160rpm with average piston speed of 150 m/min. Determine the size of the cylinder.

GIVEN DATA

Power (P) = 150KW

Piston speed (2LN) = 150m/min = $\frac{150}{60} = 2.5 \frac{m}{s}$

Speed (N) = 160rpm $160/60 = 2.7\text{ rps}$

Pressure (P1) = 1bar = 100KN/m^2

Pressure (P2) = 6bar = 600KN/m^2

$PV^{1.35} = C$, $n = 1.35$

Hence it is a polytropic process.

TO FIND

Size of the cylinder (d)?

SOLUTION

It is given that,

$$2lN = 2.5\text{m/s}$$

$$l = \frac{2.5}{2 \times 2.7}$$

$$l = 0.4629\text{m}$$

$$\text{since } V_1 = V_s = \frac{\pi}{4} d^2 l$$

$$V_1 = V_s = \frac{\pi}{4} d^2 (1.4629)$$

$$V_1 = 0.3635d^2$$

We know that,

$$\text{Power (P)} = 2 \times W \times N \quad (\text{for double acting})$$

For polytropic process, work done (W) is

$$W = \frac{n}{n-1} (P_1 V_1) \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$W = \frac{1.35}{1.35-1} (100 \times 0.3635d^2) \left[(6)^{\frac{1.35-1}{1.35}} - 1 \right]$$

$$W = 82.899 d^2$$

$$\text{Power (P)} = 2 \times W \times N$$

$$150 = 2 \times 82.899 d^2 \times 2.7$$

$$d^2 = 0.3350$$

$$d = 0.57M$$

2. A single stage single acting reciprocating air compressor is required to handle 30m^3 of free air per hour measured at 1 bar . the delivery pressure is 6.5 bar and the speed is 450 r.p.m allowing volumetric efficiency of 75%;an isothermal efficiency of 76% and mechanical efficiency of 80% Find the indicated mean effective pressure and the power required the compressor

GIVEN DATA

Volume	$V_1 = 30\text{m}^3$
Pressure	$P_1 = 1 \text{ bar} , P_2 = 6.5 \text{ bar}$
Speed	$N = 450 \text{ r.p.m}$
Volumetric efficiency	$\eta_v = 75\%$
Isothermal efficiency	$\eta_i = 76\%$
Mechanical efficiency	$\eta_m = 80\%$

TO FIND

- The indicated mean effective pressure
- The power required to drive the compressor

SOLUTION

Indicted Mean Effective Pressure

We know that isothermal work done

$$= 2.3V_1P_1 \log \left[\left(\frac{P_2}{P_1} \right) \right]$$



$$=2.3 \times 10^5 \times 30 \log \left[\left(\frac{6.5}{1} \right) \right]$$

$$=5609 \times 10^3 \text{ J/h}$$

And indicated work done = $\frac{\text{Isothermal work done}}{\text{Isothermal efficiency}}$

$$= \frac{5609}{0.76} = 7380 \text{ KJ/h}$$

We know that swept volume of the piston

$$V_s = \frac{\text{volume of free air}}{\text{volumetric efficiency}} = \frac{30}{.75}$$

$$= 40 \text{ m}^3/\text{h}$$

Indicated mean effective pressure $p_m = \frac{\text{indicated work done}}{\text{swept volume}} = \frac{7380}{40}$

$$= 184.5 \text{ kJ/m}^3$$

$$= 184.5 \text{ KN/m}^2$$

The power required to drive the compressor

We know that work done by the compressor = $\frac{\text{indicated work done}}{\text{mechanical efficiency}}$

$$= \frac{7380}{.8}$$

$$= 9225 \text{ KJ/h}$$

Therefore the power required to drive the compressor = $\frac{9225}{3600}$

$$=2.56\text{KW}$$

RESULT

Indicated mean effective pressure $p_m=184.5\text{KN/m}^2$

The power required to drive the compressor $=2.56\text{KW}$

3. A two stages, single acting air compressor compresses air to 20bar. The air enters the L.P cylinder at 1bar and 27°C and leaves it at 4.7bar. The air enters the H.P. cylinder at 4.5bar and 27°C . the size of the L.P cylinder is 400mm diameter and 500mm stroke. The clearance volume in both cylinder is 4% of the respective stroke volume. The compressor runs at 200rpm, taking index of compression and expansion in the two cylinders as 1.3, estimate 1. The indicated power required to run the compressor; and 2. The heat rejected in the intercooler per minute.

GIVEN DATA

Pressure (P4)= 20bar

Pressure (P1) = 1bar = $1 \times 10^5 \text{ N/m}^2$

Temperature (T1) = $27^\circ\text{C} = 27+273 = 300\text{K}$

Pressure (P2) = 4.7bar

Pressure (P3) = 4.5bar

Temperature (T3) = $27^\circ\text{C} = 27+273 = 300\text{K}$

Diameter (D1) = 400mm 0.4m

Stroke (L1) = 500mm = 0.5m

$$K = \frac{v_{c1}}{v_{s1}} = \frac{v_{c3}}{v_{s3}} = 4\% = 0.04$$

$$N = 200 \text{ rpm ; } n = 1.3$$

TO FIND

Indicated power required to run the compressor

SOLUTION

We know the swept volume of the L.P cylinder

$$\begin{aligned} v_{s1} &= \frac{\pi}{4} (D_1)^2 L_1 = \frac{\pi}{4} (0.4)^2 0.5 \\ &= 0.06284 \text{ m}^3 \end{aligned}$$

And volumetric efficiency,

$$\begin{aligned} \eta_v &= 1 + \frac{K}{n} - K \left(\frac{P_2}{P_1} \right)^{\frac{1}{n}} \\ &= 1 + 0.04 - 0.04 \left(\frac{4.7}{1} \right)^{\frac{1}{1.3}} \\ &= 0.9085 \text{ or } 90.85\% \end{aligned}$$

Volume of air sucked by air pressure compressor,

$$\begin{aligned} v_1 &= v_{s1} \times \eta_v = 0.06284 \times 0.9085 = 0.0571 \frac{\text{m}^3}{\text{stroke}} \\ &= 0.0571 \times N_w = 0.0571 \times 200 = 11.42 \text{ m}^3/\text{min} \end{aligned}$$

And volume of air sucked by H.P compressor,

$$v_3 = \frac{P_1 V_1}{P_3} = \frac{1 \times 11.42}{4.5} = 2.54 \frac{m^3}{min}$$

We know that indicated work done by L.P compressor,

$$\begin{aligned} W_L &= \left(\frac{n}{n-1} \right) P_1 v_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] \\ &= \left(\frac{1.3}{1.3-1} \right) 1 \times 10^5 \times 11.42 \left[\left(\frac{4.7}{1} \right)^{\frac{1.3-1}{1.3}} - 1 \right] \\ &= 2123.3 \times 10^3 \text{ J/min} = 2123.3 \text{ KJ/min} \end{aligned}$$

And indicated work done by H.P compressor,

$$\begin{aligned} W_H &= \left(\frac{n}{n-1} \right) P_3 v_3 \left[\left(\frac{P_4}{P_3} \right)^{\frac{n-1}{n}} - 1 \right] \\ &= \left(\frac{1.3}{1.3-1} \right) 4.5 \times 10^5 \times 2.54 \left[\left(\frac{4.20}{4.5} \right)^{\frac{1.3-1}{1.3}} - 1 \right] \\ &= 2034.5 \times 10^3 \text{ J/min} = 2034.5 \text{ KJ/min} \end{aligned}$$

Total indicated work done by the compressor,

$$W = W_L + W_H = 2123.3 + 2034.5 = 4157.8 \text{ KJ/min}$$

Indicated power required to run the compressor

$$= 4157.8 / 60 = 69.3 \text{ KW}$$

4.9 TWO MARK UNIVERSITY QUESTIONS:**Part-A (2 Marks)**

1. What is meant by single acting compressor?
2. What is meant by double acting compressor?
3. What is meant by single stage compressor?
4. What is meant by multistage compressor?
5. Define isentropic efficiency
6. Define mean effective pressure. How is it related to in power of an I.C engine.
7. What is meant by free air delivered?
8. Explain how flow of air is controlled in a reciprocating compressor?
9. What factors limit the delivery pressure in reciprocating compressor?
10. Name the methods adopted for increasing isothermal efficiency of reciprocating air compressor.
11. Why clearance is necessary and what is its effect on the performance of reciprocating compressor?
12. What is compression ratio?
13. What is meant by inter cooler?

4.10 UNIVERSITY ESSAY QUESTIONS:

Part-B (16 Marks)

1. Drive an expression for the work done by single stage single acting reciprocating air compressor. (16)
2. Drive an expression for the volumetric efficiency of reciprocating air compressors (16)
3. Explain the construction and working of a root blower (16)
4. Explain the construction and working of a centrifugal compressor (16)
5. Explain the construction and working of a sliding vane compressor and axial flow compressor.(16)
6. A single stage single acting air compressor is used to compress air from 1 bar and 22°C to 6 bar according to the law $PV^{1.25} = C$. The compressor runs at 125 rpm and the ratio of stroke length to bore of a cylinder is 1.5. If the power required by the compressor is 20 kW, determine the size of the cylinder. (16)
7. A single stage single acting air compressor is used to compress air from 1.013 bar and 25°C to 7 bar according to law $PV^{1.3} = C$.The bore and stroke of a cylinder are 120mm and 150mm respectively. The compressor runs at 250 rpm .If clearance volume of the cylinder is 5% of stroke volume and the mechanical efficiency of the compressor is 85%, determine volumetric efficiency, power, and mass of air delivered per minute. (16)



8. A two stage single acting air compressor compresses 2m³ air from 1 bar and 20° C to 15 bar. The air from the low pressure compressor is cooled to 25° C in the intercooler. Calculate the minimum power required to run the compressor if the compression follows $PV^{1.25}=C$ and the compressor runs at 400 rpm. (16)